

Principles and Rules for Translating Retirement Objectives into Strategies

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Abstract

This article sets out principles and decision rules for setting appropriate drawdown and investment strategies during retirement given an individual's objectives and risk tolerance. In particular, we highlight how the suitable drawdown strategy can relate to the objective, and how annuities might be combined with other investments to help manage risk. Our aim is to offer research-based guidance to product providers, financial advisers and individuals in formulating retirement strategies.

Introduction

A joint drawdown and investment strategy is required when deploying assets during retirement. Determining suitable strategies is complicated by the fact that retirees can differ along many dimensions. This article sets out principles and associated decision rules for determining the type of strategy that might be appropriate given an individual's objectives and risk tolerance. The principles and rules are formed by interpreting academic research that models 'optimal' retirement strategies in an Australian context, including allowing for the Age Pension. We primarily refer to our own modelling of fourteen types of Australian individuals that vary by objectives (i.e. utility function), risk tolerance, balance and homeownership (Butt, Khemka and Warren, 2021). An overview of our research appears in the Appendix. We also draw on similar work by other researchers including: Kingston and Thorp (2005); Hulley et al. (2013); Iskhakov, Thorp and Bateman (2015); Andréasson, Shevchenko and Novikov (2017); Bell, Liu and Shao (2017); Andréasson and Shevchenko (2017, 2019); and Warren (2020).

We start by listing the information required to characterise objectives and preferences and translate them into an appropriate strategy. We then discuss three broad types of retirement outcomes that individuals may care about and hence will define their objectives: (1) the existence of any minimum acceptable income; (2) whether the individual either (a) targets a specific income level, or (b) aims to maximise the income extracted from their assets; and (3) any bequest motive. The type of optimal strategy that emerges from the modelling in the presence of each objective is outlined. This provides the basis for the principles and decision rules, which aim to capture the essence of the optimal strategies in a way that can be readily applied. The influence of risk tolerance² on strategy implementation is discussed before concluding.

Information required to specify an appropriate strategy

An understanding of individual objectives³ is foundational for determining suitable strategies and products. This is reinforced by the design and distribution obligations,⁴ which require the "likely objectives, financial situation and needs" of consumers to be addressed when offering financial products. Warren (2020) discusses the information needed to identify objectives and translate them into suitable retirement strategies. Exhibit 1 lists the key items,⁵ noting that our principles and rules relate to the 'Objectives' and 'Risk tolerance' elements. The items listed under 'Other key information' have no significant impact on the principles and rules, but may influence how they are implemented. For instance, household status and homeownership can affect the magnitude of any minimum acceptable income or income target, and the assets available to achieve these objectives. Nevertheless, the idea of directing the strategy toward delivering a certain income level for as long as possible still applies.

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² We do not address the issue of eliciting risk tolerance, which is a major topic in itself.

³ The academic research referred to embeds objectives within utility functions.

⁴ <https://www.legislation.gov.au/Details/C2019A00050>.

⁵ Our research in Butt, Khemka and Warren (2021) spans most of the aspects listed, except for minimum acceptable incomes, household status and precautionary savings.

Exhibit 1: Key Information Required for Designing Retirement Strategies

Objectives:

1. **Existence of any minimum acceptable income** – Refers to any minimum level of income that an individual needs to avoid poverty. Relevant only if in excess of the Age Pension, e.g. renters.
2. **Type of income objective** – Refers to *desired* income to support spending. There are two main alternatives:
 - a) **Income target** – Aim of achieving a defined level of income and hence spending during retirement, without any pronounced aspiration to spend much more. The target might be time-varying.
 - b) **Income maximisation** – Aim of extracting as much income as possible out of the available assets.
3. **Bequest motive** – Desire to leave a bequest; which might be either: (a) non-existent, (b) incidental, or (c) strong.

Risk tolerance:

4. Willingness to accept the risk of falling short of objectives in pursuit of better outcomes.

Other key information:

5. **Household status** – Whether the individual has a partner (or other dependents). Designing strategies for couples requires accounting for joint assets, combined spending and dual life expectancy.
6. **Homeownership** – Renters require more income than homeowners. The family home might also help satisfy bequest or precautionary savings motives, or provide an additional source of income.
7. **Other available assets** – Strategies should be designed around total assets available to fund retirement outcomes.
8. **Precautionary savings** – Desire to keep some funds aside for unexpected developments.⁶ May be handled by carving out an amount from the ‘available’ assets prior to specifying the retirement strategy.

Minimum acceptable income levels – *Secure any minimum with annuities, if possible*

The concept of a minimal acceptable income is underpinned by poverty avoidance. We did not model minimum acceptable outcomes in Butt, Khemka and Warren (2020). However, they are addressed in studies incorporating a consumption floor within their utility function (Kingston and Thorp, 2005; Ishakov et al., 2015; Andréasson and Shevchenko, 2019); and in Warren (2020), who imposes a very large utility penalty below the minimum. In all cases, annuities are used to avoid breaching the floor. The use of annuities in this way is intuitive. If falling below some level of income would leave an individual destitute, any chance of this occurring should be eliminated if possible. It can also be motivated by appealing to risk capacity (see Klement, 2018), which suggests hedging risk that there is no capacity to bear. The following principle and decision rule arises:

Principle – *Secure any minimum acceptable income, if possible.*

Decision rule – *Purchase sufficient annuities to secure the minimum, accounting for other available income.*

Two situations may negate the rule of purchasing annuities to secure the minimum. First is where there is virtually zero likelihood of falling below the minimum. This could be due to sufficient income support being available, e.g. Age Pension being adequate,⁷ defined benefit pensions, family support such as rich, aging parents. Alternatively, assets could be sufficiently large that there is little chance of falling below the minimum. The second situation is where available assets are insufficient to buy an annuity that secures the minimum acceptable outcome. Choices under these circumstances include either: (a) take risk in a bid to reach the minimum (e.g. invest in 100% growth assets), or (b) buy annuities to secure as much income as possible, while accepting the inevitability of being left short. The appropriate strategy in this situation depends on individual preferences.⁸

⁶ Precautionary and bequest motives may interact, as funds set aside might be used to satisfy either.

⁷ RIR (2020) highlights that non-homeowners are more likely to fall below the poverty line due to the need to pay rent.

⁸ Prospect theory (see Tversky and Kahneman, 1992) predicts the ‘gambling for redemption’ strategy; and this also emerges from our modelling. Warren (2019, 2020) discusses whether it may be more appropriate to assume increasing aversion to larger losses in certain contexts such as retirement, in which case an annuity might be purchased regardless.

Income targets – Drawdown to meet the target, and invest to secure it

A key contribution of Butt, Khemka and Warren (2021) is examining optimal strategies under a consumption (i.e. income) target. While their analysis is based around the AFSA Retirement Standards, the findings identify principles and rules that can be applied under *any* income target – be it based on a budget standard, an income replacement rate, or simply an individual spending preference. We initially convey and explain the principle that emerges, before offering decisions rules for applying that principle.

Principle – *Invest with the aim of delivering the income target for as long as possible using a combination of annuities, growth and defensive assets. Then draw a sufficient amount to deliver the income target, until available assets are exhausted.*

The principle of investing to deliver the income target for as long as possible manifests under our modelling as a growth asset allocation that is u-shaped in retirement balance, along with use of annuities to secure the target. Such a pattern can be understood as follows:

- *Low assets* – Individuals with insufficient assets to secure the target income through (say) purchasing annuities invest entirely in growth assets. This maximises the probability of attaining the target for longer, albeit with increased risk of the assets lasting for a shorter period if returns are poor.⁹ The rationale is enhanced by the strategy being directed at sustaining income through to later in retirement, thus implying a long investment horizon. The literature indicates that higher-returning, riskier assets almost dominate lower-returning, safer assets over long horizons, especially where a target exists (see Warren, 2019).
- *Sufficient assets to secure the target* – At some level of assets, the opportunity presents to secure the income target using annuities, or perhaps other defensive investments. It is optimal to take this opportunity through annuity purchases, especially for individuals with low risk tolerance (see later discussion).
- *High assets* – As assets increase, surplus funds can be allocated to growth assets to pursue additional income (or bequests), in the confidence that the target has been secured. An increase in the percentage allocation to growth assets is thus observed as balance increases beyond a certain point.

The role of annuities versus other defensive assets is worth discussing. Our modelling reveals the tendency for (real) life annuities to crowd out defensive assets such as fixed income, as they can facilitate securing the target for life. We also observe use of both immediate life annuities and deferred life annuities. Deferred life annuities tend to be favoured, with target income attained in the interim by drawing on other available¹⁰ assets. This strategy provides longevity insurance at a modest cost, and offers potential for upside if returns are high. Where there exists a meaningful risk of failing to sustain the target until the deferred annuity kicks if returns are low, some immediate life annuities might be purchased to help secure the target. Defensive asset holdings can also emerge in some situations. In particular, they can help to secure the income target before any deferred life annuity income kicks-in through providing a mechanism to dial back risk exposure if returns are low.¹¹ The use of annuities and defensive assets in these ways accord with directing the investment strategy towards delivering the target.

The drawdown principle implies drawing enough to reach the target after accounting for other income sources such as the Age Pension and any annuities, subject to the minimum drawdown rules. Once available assets are exhausted, income then declines to that arising from these other sources. As an example, if the income target is \$30,000 and the Age Pension and supplements deliver \$24,500, then \$5,500 is drawn until assets are exhausted. This drawdown strategy can be justified intuitively in the following way. First, there is only minimal benefit from drawing more than required to attain the target, especially as doing so could increase the risk of falling short later in retirement. Second, while drawing a below-target amount extends the timeframe over which the assets last, it also lowers the probability that the income will be enjoyed because of being less likely to be alive later in retirement. It is hence better to spend the target amount while able. The main caveat on the general principle arises where the target is secured, say through annuity purchases. In this situation, an option exists to either spend a bit more, or allow assets to build in support of a bequest.

⁹ This risk is heightened by the sequencing risk that emerges under the influence of drawdowns.

¹⁰ Available assets are those remaining after amounts set aside for precautionary savings and any annuity purchases.

¹¹ Our modelling allowed annuity purchase only at retirement. We expect that annuity purchases would occasionally be observed during retirement if they were available, in lieu of increasing defensive asset exposure to help protect income.

Decision rules for an income target objective:

1. *Establish the annuity purchases required to secure the target income for life, allowing for the Age Pension and other income sources* – The preference is combining a deferred life annuity with the intent to draw on other assets, with immediate life annuities considered if required to help secure the target in the interim.
2. *Purchase the required annuities, subject to not exceeding some upper limit of available assets* – Applying an upper limit on annuity purchases retains some access to capital and related flexibility; while recognising the well-known reluctance to commit entirely to annuities and the Age Pension providing a level of income assurance. We suggest an upper limit of 60%.¹²
3. *Where available assets are insufficient, scale back annuities purchases to guarantee any minimum acceptable income* – This rule applies when the annuities required to secure the income target are unaffordable.
4. *Invest remaining amount in growth assets, as far as can be tolerated* – While growth assets should be preferred, this rule recognises that growth exposure might dialled back for individuals who are uncomfortable with asset volatility: we discuss this matter under risk tolerance.

Maximising income – Frame around affordable drawdown and a level of income protection

The objective of some individuals may be to extract as much income as possible out of their assets, while managing the risk of income falling to undesirably low levels. This requires balancing the potential consequences of drawing too much earlier and/or poor investment returns, against failing to fully utilise the available assets due to not drawing enough and dying with large residual assets.¹³ In trading off these elements, academic models tend to generate optimal strategies involving time-varying drawdowns and investment strategies that combine growth assets with annuities. Annuities are directed at smoothing and limiting the downside to income; tend to crowd out other defensive assets (e.g. fixed income); and combinations of both life annuities and deferred life annuities are observed. The following principle is suggested:

Principle – *The drawdown and investment strategy should be jointly directed at converting assets into as much income as possible, while leaving behind no more assets than intended.*¹⁴

Making full use of available assets under uncertainty over investment returns and mortality requires some form of dynamic strategy. The rules below capture the essence and intent of the strategies observed in the modelling.

Decision rules under an income maximisation objective:

1. *Determine how much income to protect using annuities* – Setting the income to protect with annuities (in excess of minimum acceptable income) captures the notion of smoothing income and limiting its downside. Annuities are purchased to guarantee this income after allowing for other potential income sources such as the Age Pension. How much income to protect will be largely a function of risk tolerance (see discussion below), and we suggest subject to an upper limit of 60%.
2. *Invest remainder in growth assets, as far as can be tolerated* – Having protected a certain amount of income, growth assets should be favoured to maximise potential income over the long run, perhaps with some dialling back of growth exposure for those uncomfortable with asset volatility.
3. *Establish an affordable drawdown strategy* – The ‘affordable’ drawdown given the assets remaining after annuity purchases is estimated and initially drawn. One method is to calculate the real amount that can be sustained to a certain age given expected returns under the selected asset mix. This method is similar to the 4% rule of Bengen (1994), except that the percentage of initial balance drawn may be much higher with our estimates suggesting something nearer 8%. Alternatively, a drawdown rate might be specified that varies with age and balance, aimed at utilising the balance while adjusting in response to investment returns. The Actuaries Institute has proposed such a rule (see De Ravin et al., 2019).

¹² The maximum allocation to annuities under our modelling is 62%; which is similar to the maximum reported by Iskhakov, Thorp and Bateman (2015). Bell, Liu and Shao (2017) find utility start to decline sharply once annuitisation exceeds 60%.

¹³ RIR (2020) highlights the failure to draw down balances as common behaviour, and a problem that needs addressing.

¹⁴ The word ‘intended’ envisages any bequest motive or desire to maintain some precautionary savings.

4. *Review the drawdown strategy occasionally* – The drawdown strategy is reviewed occasionally, including re-estimating the affordable drawdown. Reasons to adjust the drawdown might include returns differing from expectations, or adjustments to the target age due to health developments.

Investment-linked annuities¹⁵ capture many features of the above rules. They may be designed to generate expected income based around what is affordable given the underlying investment strategy, along with procedures to adjust drawdowns for investment experience.¹⁶ Longevity risk can also be hedged through mortality pooling.¹⁷

Bequests – *Limit drawdowns and reduce annuities under a strong bequest motive*

Our modelling finds that a strong bequest motive has a significant impact on optimal strategies, leading to substantial reductions in drawdowns and annuity purchases and greater growth asset exposure relative to an incidental bequest motive. The principle and rules below are aimed at building up assets via restricting spending and investing in assets that build a nest-egg.

Principle – *A strong bequest motive implies restricting drawdowns to tolerable levels, and directing the investment strategy towards building assets.*

Decision rules under a strong bequest motive:

1. *Use annuities only as far as required to secure any minimum acceptable income*
2. *Drawdown only what is needed*
3. *Invest in growth assets, as far as can be tolerated*

It may be helpful to specify the drawdown strategy as an income target that is set somewhat lower than the affordable drawdown.

Risk Tolerance – *Moderates how the principles and rules are applied*

Risk tolerance moderates how the principles and rules are implemented. The discussion so far implicitly presumes low tolerance for falling short of objectives. Our modelling indicates that lower risk aversion or loss aversion is associated with lesser use of annuities, greater growth asset exposure, more volatile drawdowns, and increased willingness to accept the possibility of lower income. The following principle sums up these indications.

Principle – *An individual with high risk tolerance should reduce annuity use in favour of additional growth assets, and use the opportunity to increase their expected income (or bequest).*

For instance, annuity purchases might be limited to that required to secure any minimal acceptable income. Drawdowns might then be initially increased in line with a higher estimate of the affordable drawdown, on the understanding they will need to be lowered if investment returns are poor.

Although not addressed in our modelling, ample evidence exists that some individuals have limited appetite for asset volatility over shorter periods.¹⁸ Recognising this suggests the following additional principle:

Principle – *Set growth asset exposure at the maximum that can be tolerated.*

One way of implementing this principle may be to estimate the growth asset weight that delivers no more than an x% probability of -y% loss over 1, 2 or 3 years. Alternatively, a bucketing strategy¹⁹ might be used as a framing device to help individuals deal with short-term market fluctuations. Implications of constraining growth assets

¹⁵ For a description, see <https://www.actuaries.digital/2014/04/15/investment-linked-lifetime-annuity/> or <https://www.superguide.com.au/in-retirement/investment-linked-annuity>.

¹⁶ The literature has proposed methods for smoothing the drawdown adjustments made in response to realised investment returns, including reserving or buffer funds (for instance, see Maurer et al., 2019) and partially adjusting the income drawn (e.g. van Bilsen and Linders, 2019).

¹⁷ To date, our modelling has only examined investment-linked annuities at a rudimentary level, and has not investigated mortality pooling. We intend to explore these areas in future research.

¹⁸ Butt et al. (2018) report that 53% of members surveyed agreed with the statement: “I will accept lower average returns to avoid losses in the short term”.

¹⁹ See <https://www.superguide.com.au/in-retirement/bucket-strategy-solution-retirement-income-plan>.

may include: (a) low-balance individuals face a reduced probability of attaining an income target; (b) reduced scope for income to exceed any income target; (c) lower affordable drawdown levels; and (d) lower bequests.

Final Thoughts

This article draws on research by ourselves and others to propose principles and decision rules to guide the formation of retirement strategies under differing objectives. The principles and rules do not in themselves deliver the specifics for strategies: we leave this for product designers and financial advisers. Nevertheless, they do indicate the broad type of strategy that may be suitable given objectives and risk tolerance. We believe that our principles and rules are sensible, but realise that they are sometimes at odds with observed behaviours such as anchoring on minimum drawdown rules and widespread use of fixed income rather than annuities for defensive exposure. We offer them as way to improve on existing practices, noting they are rooted in rigorous modelling.

APPENDIX

Scope of the Analysis in Butt, Khemka and Warren (2021)

Our modelling in Butt, Khemka and Warren (2021) identifies optimal investment and drawdown strategies for seven ‘cameo’ individuals that differ by balance at retirement (\$200,000, \$500,000 and \$800,000), objectives (i.e. utility function) and homeownership. We impose strong and weak loss or risk aversion under each cameo, thus generating analysis for 14 individual types. Investment strategies are formed by allocating the balance at retirement between an account-based pension, a real immediate life annuity and a real deferred life annuity; and then dynamically varying the growth/defensive weight within the account-based pension and drawdowns until death. The analysis allows for both the Age Pension and related supplements, as well as the minimum drawdown rules.

Our baseline formulation maximises expected utility from consumption during retirement plus bequests under two utility functions. The first of ‘loss aversion utility’ is based around the cumulative prospect theory of Tversky and Kahneman (1992). It involves evaluating consumption relative to targets based on either ASFA Modest or ASFA Comfortable (with housing cost adjustments for renters), and provides insights into optimal strategies for individuals who have an income target of any description. The second ‘constant relative risk aversion’ or ‘power’ utility function is defined over raw levels of consumption and bequests, and provides insights into optimal strategies for individuals who aim to maximise the income extracted from their available balance while managing risk. Bequests are treated as incidental gains under the baseline analysis, in the sense that they are evaluated as additional income, but additional analysis is conducted under a strong bequest motive. Expected utility is converted into certainty equivalents to judge the economic impacts.

Dynamic strategies delivered under dynamic programming can be difficult to implement in practice. Hence, we subsequently analysed some simple rules of thumb for both the investment and drawdown strategy, evaluating them against the optimal strategy based on the utility that they generate. Our rule of thumb analysis is not formally written up, but results can be provided on request. The analysis provided us with additional insight into the preferred strategies under differing circumstances and assumptions. It indicates that the wide range of individual circumstances during retirement cannot be adequately catered for by one-size-fits-all rules of thumb. This finding helped motivate the principles and decision rules appearing in this article, which we argue provide more flexibility than imposing particular rules of thumb.

Exhibit 2 summarises the scope of the research, including listing the rules of thumb that we tested and the sensitivity analysis undertaken. It highlights how we covered a lot of ground by examining seven cameos, varying degrees of risk tolerance and bequest motives, differing input assumptions and the application of various ‘rule of thumb’ strategies. The analysis yielded considerable insight to what strategies are more suitable under certain circumstances, allowing us to identify many of the principles and decision rules outlined in this article.

Exhibit 2: Scope of the Modelling Undertaken

Consumption / Income Target Focused (Loss Aversion Utility)				
Cameo	Balance at Retirement	Home-owner	Consumption / Income Target	Rules of Thumb Applied
1.	\$200,000	No	ASFA Modest + Rent Adjustment	<p>Investment</p> <ol style="list-style-type: none"> No annuities purchased Annuities purchased – life annuity sufficient to guarantee ASFA Modest, plus deferred life annuity sufficient to guarantee ASFA Comfortable after age 85. (NB: Deferred annuity not purchased at \$200,000 balance.) Growth/defensive mix in account-based pension: 100/0; 70/30; 30/70; bucketing (4-years of drawdowns invested in defensive assets) <p>Drawdowns from account-based pension</p> <ol style="list-style-type: none"> Draw no more than enough to meet consumption target regardless, subject to minimum drawdown rules Draw enough to meet consumption target plus additional fixed amount that maximises expected utility
2.	\$200,000	Yes	ASFA Modest	
3.	\$500,000	No	ASFA Comfortable + Rent Adjustment	
4.	\$500,000	Yes	ASFA Comfortable	
5.	\$800,000	Yes	ASFA Comfortable	
Utility Parameters		<ul style="list-style-type: none"> Loss aversion: <ul style="list-style-type: none"> Strong (Blake et al., 2013) Weak (Tversky and Kahneman, 1992) Weak bequest motive 		
Consumption / Income Level Focused (Constant Relative Risk Aversion Utility)				
Cameo	Balance at Retirement	Home-owner	Consumption / Income Target	Rules of Thumb Applied
6.	\$500,000	Yes	None	<p>Investment</p> <ol style="list-style-type: none"> No annuities purchased Annuities purchased, applying 10% of balance to life annuity and 10% to deferred life annuity Growth/defensive mix in account-based pension: 100/0; 70/30; 30/70; bucket (4-years of drawdowns invested in defensive assets) <p>Drawdowns from account-based pension</p> <ol style="list-style-type: none"> Minimum drawdown only Draw x% of starting account-based pension balance; with x% located such that expected utility is maximised Actuaries Institute rule (De Ravin et al., 2019), where the percentage of balance drawn depends on age and balance Investment Linked Life Annuity, with drawdown adjusted for realised return vs. target return
7.	\$800,000	Yes	None	
Utility Parameters		<ul style="list-style-type: none"> Risk aversion: <ul style="list-style-type: none"> Strong (coefficient of risk relative risk aversion of 5) Weak (coefficient of risk relative risk aversion of 2) Weak bequest motive 		
Available Assets		Key Retirement System Assumptions		Sensitivity Tests
<ul style="list-style-type: none"> Account-Based Pension Real compound return: <ul style="list-style-type: none"> Growth 5% Defensive 1% Immediate life annuity (CPI-adjusted) Deferred life annuity, starting from age 85 (CPI-adjusted) <i>(Annuity pricing supplied by Challenger)</i> 		<ul style="list-style-type: none"> Age Pension available; growing at +1.5% real (i.e. average weekly earnings); assets and income tests applied Pension supplements available; growing at 0% real (i.e. in line with CPI) AFSA targets applied for both pre- and post-age 85; projected targets equal Age Pension plus current real difference between ASFA Standards and Age Pension Rents and home prices grow at 1.5% (i.e. long-term labour productivity rate) Minimum drawdown rules imposed 		<ol style="list-style-type: none"> Strong bequest motive (parameterised as per Lockwood, 2018) Lower real returns <ul style="list-style-type: none"> Growth 3% (-1% and -2% vs. baseline) Defensive 0% (-1% vs. baseline) Known time of death at age 97; equals age of 10% probability of survival; effectively removes mortality weighting of outcomes from analysis

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